QUARTERLY EXAMINATION - 2024

MODEL PAPER - I

STD: XII **Marks: 90**
Maths **Time: 3 Hrs**

PART - I $(20 \times 1 = 20)$

Note: (i) All Questions are compulsory
(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer

1. If $|\text{adj}(\text{adj } A)| = |A|^9$, then the order of the square matrix A is
(a) 3 (b) 4 (c) 2 (d) 5

2. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ then $9I - A =$
(a) $2A^{-1}$ (b) $3A^{-1}$ (c) A^{-1} (d) $\dfrac{A^{-1}}{2}$

3. If $A^T \cdot A^{-1}$ is symmetric, then $A^2 =$
(a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

4. The system of equations $ax + y + z = 0$, $x + by + z = 0$, $x + y + cz = 0$ has a non-trivial solution then $\dfrac{1}{1-a} + \dfrac{1}{1-b} + \dfrac{1}{1-c} =$
(a) 1 (b) -1 (c) 0 (d) 2

5. Which of the following are true of properties of complex conjugates
(i) z is real if and only if $z = \bar{z}$
(ii) z is purely imaginary if and only if $z = -\bar{z}$
(iii) $\overline{(z^n)} = (\bar{z})^n$ when n is integer
(iv) $\text{Re}(z) = \dfrac{z + \bar{z}}{2}$ and $\text{Im}(z) = \dfrac{z - \bar{z}}{2}$
(a) (i) (ii) (iii) (b) (i) (ii) (iv) (c) (i) (ii) (d) All of these

6. The solution of the equation $|z| - z = 1 + 2i$ is
(a) $\dfrac{3}{2} - 2i$ (b) $\dfrac{-3}{2} + 2i$ (c) $2 - \dfrac{3i}{2}$ (d) $2 + \dfrac{3i}{2}$

7. If $\omega \neq 1$ is a cube root of unit and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$ then k is equal to
(a) 1 (b) -1 (c) $\sqrt{3}\, i$ (d) $-\sqrt{3} i$

8. The polynomial $x^3 - kx^2 + 9x$ has three real roots if and only if, k satisfies

(a) $|k| \le 6$ (b) $k = 0$ (c) $|k| > 6$ (d) $|k| \ge 6$

9. The polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has maximum number of real, imaginary roots are

(a) 6, 3 (b) 3, 6 (c) 5, 4 (d) 4, 5

10. Which of the following one is not a periodic function with period 2π radians

(a) $\sin x$ (b) $\cos x$ (c) $\tan x$ (d) $\operatorname{cosec} x$

11. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$ then $\cos^{-1} x + \cos^{-1} y$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

12. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) $[1, 2]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) $[-1, 0]$

13. $\sin(\tan^{-1} x), |x| < 1$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

14. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

(a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

15. If $x + y = k$ is a normal to the parabola $y^2 = 12x$ then the value of k is

(a) 3 (b) -1 (c) 1 (d) 9

16. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) $2ab$ (b) ab (c) $\sqrt{ab}$ (d) $\frac{a}{b}$

17. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}\ \vec{c}\ \vec{b}]$ is equal to

(a) 2 (b) -1 (c) 1 (d) 0

18. If $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = 64$ then $[\vec{a}\ \vec{b}\ \vec{c}]$ is

(a) 32 (b) 128 (c) 0 (d) 8

19. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{k}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$ then k is equal to

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$ (c) $\frac{5}{3}$ (d) $\frac{2}{3}$

20. If the directions of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

(a) $c = \pm 3$ (b) $c = \pm\sqrt{3}$ (c) $c > 0$ (d) $0 < c < 1$

PART - II **($7 \times 2 = 14$)**

Note: (i) Answer any 7 Questions

(ii) Q.No: 30 is compulsory

21. If adj $A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ find A^{-1}

22. If $z = r(\cos\theta + i\sin\theta)$ then prove that $z^{-1} = \frac{1}{r}(\cos\theta - i\sin\theta)$

23. If $z_1 = 3 - 2i$ and $z_2 = 6 + 4i$ find $\frac{z_1}{z_2}$

24. Solve the equation $x^4 - 14x^2 + 45 = 0$

25. Is $\cos^{-}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.

26. Find the value of $\sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$

27. Obtain the equation of the circle for which (3, 4) and (2, − 7) are the ends of a diameter.

28. The volume of the parallelopiped whose co-terminous edges are $7\vec{i} + a\vec{j} - 3\vec{k}$, $\vec{i} + 2\vec{j} - \vec{k}$, $-3\vec{k} + 7\vec{j} + 5\vec{k}$ is 90 cubic units. Find the value of 'a'.

29. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ prove that $[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

30. For what value of $\vec{k}$, $(k + 9)x^2 + (k + 1)x + 1 = 0$ has no real roots.

PART - III **($7 \times 3 = 21$)**

Note: (i) Answer any 7 Questions

(ii) Q.No: 40 is compulsory

31. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

32. Solve by Cramer's rule $5x - 2y + 16 = 0$; $x + 3y - 7 = 0$

33. Write in polar form $2 + i\,2\sqrt{3}$

34. Find the square root of $-11 - 60\sqrt{-1}$

35. Prove that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1 - x^2}}$, $-1 < x < 1$

36. Determine whether $x + y - 1 = 0$ is the equation of a diameter of the circle $x^2 + y^2 - 6x + 4y + c = 0$ for all values of c.

37. Find the equation of the hyperbola. Given centre (2, 1) one of the foci (8, 1) and corresponding directrix $x = 4$

38. Prove by vector method than an angle in a semi-circle is a right angle.

39. Show that the lines $\vec{r} = (6\vec{i} + \vec{j} + 2\vec{k}) + s(\vec{i} + 2\vec{j} - 3\vec{k})$ and $\vec{r} = (3\vec{i} + 2\vec{j} - 2\vec{k}) + t(2\vec{i} + 4\vec{j} - 5\vec{k})$ are skew lines and hence find the shortest distance between them.

40. Solve: $\tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{4}$

PART - IV **($7 \times 5 = 35$)**

Note: Answer all Questions

41. (a) If the system of equations $px + by + cz = 0$; $ax + qy + cz = 0$; $ax + by + rz = 0$ has a non-trivial solution and $p \neq a, q \neq b, r \neq c$ prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$

(or)

(b) A bridge has a parabolic such that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6m from the centre on either sides.

42. (a) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ then show that $x^2 + y^2 = 1$

(or)

(b) Solve the equation $(2x-3)(6x-1)(3x-2)(x-2) - 5 = 0$

43. (a) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135 = 0$ if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

(or)

(b) By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$.

44. (a) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$ show that $x^2 + y^2 + z^2 + 2xyz = 1$

(or)

(b) Identify the type of conic and find centre, foci, vertices and directrices of $9x^2 - y^2 - 36x - 6y + 18 = 0$

45. (a) If $ax^2 + bx + c$ is divided by $x + 3$, $x - 5$ and $x - 1$ the remainders are 21, 61 and 9 respectively. Find a, b, c (use Gaussian elimination method) (or)

(b) Find the non-parametric and Cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$

46. (a) Find all cube roots of $\sqrt{3} + i$

(or)

(b) Find the number of solution of the equation $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1}(3x)$

47. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1} and hence solve the system of linear equations $2x - 3y + 52 = 11,\ 3x + 2y - 4z = -5;\ x + y - 2z = -3$

(or)

(b) Derive the equation of the plane in intercept form.

QUARTERLY EXAMINATION - 2024

MODEL PAPER - II

STD: XII **Marks: 90**
Maths **Time : 3 Hrs**

PART - I $(20 \times 1 = 20)$

Note: (i) All Questions are compulsory

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer

1. If A is a square matrix of order n, then which of the following one is not true.
   (a) If A has an inverse, then it is unique.
   (b) A^{-1} exists if and only if A is non-singular
   (c) If A is a singular matrix then A^{-1} is zero
   (d) A is non-singular then $A^{-1} = \frac{1}{|A|} \text{adj } A$

2. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -1 \end{bmatrix}$ is the adjoining of 3×3 matrix A and $|A| = 4$, then x is
   (a) 15 (b) 12 (c) 14 (d) 11

3. If $\rho(A) = \rho(A \mid B)$ then the system $AX = B$ of linear equations is
   (a) consistent and has a unique solution (b) consistent
   (c) consistent has infinitely many solution (d) inconsistent

4. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is
   (a) a (b) 1 (c) -1 (d) i

5. The principal argument of $(\sin 40° + i \cos 40°)^5$ is
   (a) $-110°$ (b) $-70°$ (c) $70°$ (d) $110°$

6. The value of $\left[\frac{-1 + i\sqrt{3}}{2}\right]^{100} + \left[\frac{-1 - i\sqrt{3}}{2}\right]^{100}$ is
   (a) 2 (b) 0 (c) -1 (d) 1

7. If $2i - \sqrt{3}$ is one root of a polynomial equation, then another root is
   (a) $2i + \sqrt{3}$ (b) $-2i + \sqrt{3}$ (c) $-\sqrt{3} - 2i$ (d) $\sqrt{3}$

8. A zero of $x^3 - 64$ is
   (a) 0 (b) 4 (c) $4i$ (d) -4

9. The number of positive roots of the polynomial $\sum_{j=0}^{n} {}^nC_r (-1)^r x^r$ is

(a) 0 (b) n (c) $< n$ (d) r

10. The value of $\sin^{-1}(\cos x), 0 \le x \le \pi$ is

(a) $\pi - x$ (b) $x - \frac{\pi}{2}$ (c) $\frac{\pi}{2} - x$ (d) $\pi + x$

11. If the function $f(x) = \sin^{-1}(x^2 - 3)$ then x belongs to

(a) $[-1, 1]$ (b) $[\sqrt{2}, 2)$ (c) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ (d) $[2, -2] \cup [\sqrt{2}, -\sqrt{2}]$

12. The principal value of $\sin^{-1}(2)$ is

(a) $\frac{\pi}{4}$ (b) $\frac{-\pi}{4}$ (c) 2 (d) does not exists

13. The circle with length of major axis as diameter is called

(a) Auxiliary circle (b) Incirle (c) Real circle (d) Imaginary circle

14. __________ tangent can be drawn to a parabola from an external point on the plane.

(a) 1 (b) 2 (c) 4 (d) many

15. The equation of the normal to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ which is parallel to the line $2x + 4y = 3$ is

(a) $x + 2y = 3$ (b) $x + 2y + 3 = 0$ (c) $2x + 4y + 3 = 0$ (d) $x - 2y + 3 = 0$

16. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

17. If $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ then the value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is

(a) $|\vec{a}|\ |\vec{b}|\ |\vec{c}|$ (b) $\frac{1}{3}|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ (c) 1 (d) -1

18. Consider the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ such that $(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{b}) = \vec{0}$. Let P_1 and P_2 be the planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is

(a) $0°$ (b) $45°$ (c) $60°$ (d) $90°$

19. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is

(a) $\frac{\sqrt{7}}{2\sqrt{2}}$ (b) $\frac{7}{2}$ (c) $\frac{\sqrt{7}}{2}$ (d) $\frac{7}{2\sqrt{2}}$

20. If $2\vec{i} - \vec{j} + 3\vec{k}, 3\vec{i} + 2\vec{j} + \vec{k}$ and $\vec{i} + m\vec{j} + 4\vec{k}$ are coplanar, then value of m is

(a) 2 (b) -2 (c) 3 (d) -3

PART - II **$(7 \times 2 = 14)$**

Note: (i) Answer any 7 Questions

(ii) Q.No: 30 is compulsory

21. If $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$ find $(AB)^{-1}$.
22. Obtain the Cartesian form of the locus of $z = x + iy$ given $Im\,[(1 - i)\,z + 1] = 0$
23. Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}\,i$ as a root.
24. Find the value of $\cos^{-1}\left[\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right]$
25. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x - 1) < \pi$ holds?
26. Find the centre and radius of the circle $x^2 + y^2 + 6x - 4y + 4 = 0$
27. Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x + 2y + 3 = 0$
28. With usual notation, in any triangle ABC prove by vector method $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
29. Find the angle between the straight line $\vec{r} = (2\vec{i} + 3\vec{j} + \vec{k}) + t\,(\vec{i} - \vec{j} + \vec{k})$ and plane $2x - y + z = 5$
30. Express $\frac{(1 + i)\,(1 - 2i)}{1 + 3i}$ in rectangular form.

PART - III **$(7 \times 3 = 21)$**

Note: (i) Answer any 7 Questions

(ii) Q.No: 40 is compulsory

31. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal hence find A^{-1}
32. Solve $x + 2y + 3z = 0$; $3x + 4y + 4z = 0$; $7x + 10y + 12z$
33. The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If $v = 3 - 4i$ and $w = 4 + 3i$ find u in rectangular form.
34. If $|\,z\,| = 2$ show that $3 \le |\,z + 3 + 4i\,| \le 7$.
35. Solve the equations. $12x^3 + 8x = 29x^2 - 4$
36. Find the value of $\sin^{-1}(-1) + \cos^{-1}(1/2) + \cot^{-1}(2)$
37. Find the equation of the tangent and normal to the circle $x^2 + y^2 - 6x + 6y - 8 = 0$ at (2, 2).
38. Can you draw a plane through the given two lines?

    Justify your answer $\vec{r} = (\vec{i} + 2\,\vec{j} - 4\,\vec{k}) + t\,(2\vec{i} + 3\vec{j} + 6\vec{k})$ and $\frac{x - 3}{-2} = \frac{y - 3}{3} = \frac{z + 5}{8}$
39. Find the points where the straight line passes through (6, 7, 4) and (8, 4, 9) cuts the xz and yz planes.
40. A satellite is travelling around the earth is an elliptical orbit having the earth at a focus and of eccentricity 1/2. The shortest distance that the satellite gets to the earth is 400 km. Find the longest distance that the satellite gets from the earth.

PART - IV **$(7 \times 5 = 35)$**

Note: Answer all Questions

41. (a) If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = 3\vec{i} + 5\vec{j} + 2\vec{k}$ $\vec{c} = -\vec{i} - 2\vec{j} + 3\vec{k}$ verify that $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} - \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$

(or)

(b) A semi-elliptical arch way over a one way road has height of 3m and width of 12m. The truck has a width of 3m and a height of 2.7 m. Will the truck clear the opening of the arch way.

42. (a) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$ show that $x + y + z = xyz$

(or)

(b) Solve: $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

43. (a) If $2 \cos \alpha = x + \frac{1}{x}$ and $2 \cos \beta = y + \frac{1}{y}$ show that

(i) $xy - \frac{1}{xy} = 2i \sin(\alpha + \beta)$ (ii) $x^m y^n + \frac{1}{x^m y^n} = 2 \cos(m\alpha + n\beta)$

(or)

(b) Find the value of k which the equation $kx - 2y + z = 1$, $x - 2ky + z = -2$; $x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution

44. (a) Solve: $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

(or)

(b) Find the parametric vector non-parametric vector and cartesian form of the equations of the plane passing through the points $(3, 6, -2)$. $(-1, -2, 6)$ and $(6, 4, -2)$

45. (a) Solve by using Cramer's rule $\frac{1}{x} + \frac{2}{y} - \frac{1}{z} = 1$; $\frac{2}{x} + \frac{4}{y} + \frac{1}{z} = 5$; $\frac{3}{x} - \frac{2}{y} - \frac{2}{z} = 0$

(or)

(b) Find the equation of the circle passing through the points $(1, 1)$ $(2, -1)$ and $(3, 2)$.

46. (a) By using Gaussian elimination method balance the chemical reaction equation $C_2H_6 + O_2 \rightarrow H_2O + CO_2$

(or)

(b) Show that the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar. Also find the plane containing these lines.

47. (a) The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Determine the equation of the hyperbola if its eccentricity is 2.

(or)

(b) P represents the variable complex number z, find the locus of P if $\text{Re}\left(\frac{z+1}{z+i}\right) = 1$.